PHYSICS

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FREQUENCY ANALYSIS DURING NON-STATIONARY INTERACTION OF TWO-LAYER VISCOSE ELASTIC SHELL WITH VISCOSE LIQUID

*Abstract. **The Purpose of the Study** is to study the relationship between frequency and wave number using the equation of torsional vibrations, in particular, in the interaction of a two-layer viscoelastic shell with the outer layer of the viscoelastic body and the inner layer of the viscoelastic body, which is elastic. **The Research Methodology.** To study the problem and formulate conclusions, methods of analysis, description and synthesis of data were used. **The Scientific Novelty.** A mathematical model of physical nonlinear torsional oscillations of circular conical shell and stern was developed taking into account the physical nonlinear relations of H. Kauder. **The Conclusion.** As equations of torsional vibrations of a two-layer cylindrical shell, the equations derived by the authors in their previous works for the viscoelastic case were obtained, from which frequency equations were obtained for the case when the surface of the shell is under stress -free, and they were solved using the Maple 17 program. The graphs were analyzed for cases with different materials and thicknesses of the edge and middle layers.*

Keywords. Multilayer cylindrical shell, filler layer, torsional vibrations, frequency, wave number.

The Problem Statement. In various fields of technology, round cylindrical shells are used in pipelines, car bodies, spacecraft, aircraft and shipbuilding, and building structures. However, the analysis of the dynamic properties of cylindrical shells is more complicated. In this regard, the issue of dynamic boundary conditions for non-stationary vibrations of circular cylindrical shells is important.

The Analysis of Sources and Recent Researches. Aviscoelastic body rotating around the axis of symmetry with a constant angular velocity was considered (Khudoynazarov, 2016) in a cylindrical coordinate system. In this paper, the relationship between stress and strain is considered in the Boltzmann-Volterra form. The stress state of a round plate made of an elastoplastic material was studied (Khalmuradov, Khudoynazarov, Nishanov, 2022). It is assumed that the relationship between displacement and deformation is geometrically nonlinear. A mathematical model of physical nonlinear torsional oscillations of circular conical shell
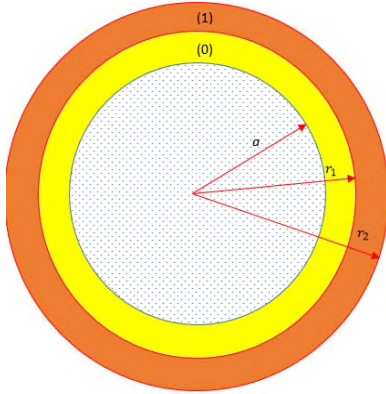
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and stern was developed taking into account the physical nonlinear relations of H. Kauder. The problem of torsional vibration of a rod with a variable radius was solved by reducing the equation of motion of an elastic body into two (homogeneous and non-homogeneous) equations. From the general solutions of the resulting equations, an algorithm is proposed that allows determining the stress-deformation state of arbitrary points of an elastic body. The equations of longitudinal vibration of a circular cylindrical elastic shell filled with Boundary problems of longitudinal-radial vibrations are studied in (Botogova, Mikhasev, Korchevskaya, 2006). Boundary and initial conditions for various types of supports allowed to consider the shell as a three-dimensional body. Based on the Kirchhoff-Love theory, a mathematical model of the problem of vibration of thin-walled structural elements was built (Filippov, Kudainazarov, 1990), (Khalmuradov, Ismoilov, 2020).) Classical and improved theories of longitudinal-radial vibrations of circular cylindrical elastic shell were studied (Khalmuradov, Yalgashev, (2020), and the frequency analysis using these equations was carried out based on the theories of Kirchhoff-Love, Hermann-Mirsky and Filippov-Khudoynazarov. Specific torsional vibrations of a three-layer cylindrical shell with a symmetrical structure have been studied.

Below, the frequency equations are derived from the equations of torsional vibrations derived from the non-stationary interaction of a two-layer cylindrical shell with a viscous fluid, with zero external loads. The graphs of the relationship between the frequency and the wave number are derived for the case where the outer layer of the two-layer shell is viscoelastic and the inner layer is elastic.

As a round-cylindrical two-layer shell, its inner layer is elastic (steel, aluminum), and its outer layer is a viscous elastic body (EDM-6).

The Purpose of the Article is to present the results of considering the problem of dynamic boundary conditions for non-stationary vibrations of circular cylindrical shells.

The Main Material Statement. We consider the shell in the (r, θ, z)-cylindrical coordinate system. In this case, the Oz axis is directed along the axis of symmetry of the shell. We number the layers as shown in Figure 1. Here, a is the inner radius of the shell, and r_1 and r_2 are the inner and outer radii of the outer layer. Below, the special torsional vibrations of the non-stationary interaction of a two-layer viscoelastic cylindrical shell with a viscous fluid are studied. To do this, we consider the case of special torsional vibrations of the shell. We use the equations presented in the article as the basic vibration equations. In order to study the specific vibrations of the shell, it is necessary to make the external forces equal to zero in the system of equations.

1-Figure. Two-layer cylindrical shell

$$\frac{r_1^2}{r_2^2}\left[1 + \frac{r_2^2 - r_1^2}{4}\lambda_1 + \frac{r_1^2\left(r_2^2 - r_1^2\right)}{16}\lambda_1^2\right]\cdot\left[\frac{r_1^2}{4}\lambda_0 U_{\theta,0}^{(0)} + \right.$$

$$\left. + \xi\left(\frac{1}{2}(\lambda_0 - \frac{4}{r_1^2}) + \frac{r_1^2}{8}(\ln\frac{r_1}{\xi} - \frac{1}{4})\lambda_0^2\right)U_{\theta,1}^{(0)}\right] = 0 \quad (1)$$

$$\left(\frac{a^2}{4}\lambda_0 - \frac{\mu' a^2}{6}\tilde{R}_{\mu 0}^{-1}q\delta_0\right)U_{\theta,0}^{(0)} + \xi\left(\frac{1}{2}(\lambda_0 - \frac{4}{a^2}) + \frac{a^2}{8}(\ln\frac{a}{\xi} - \frac{1}{4})\lambda_0^2 - \right.$$

$$\left. - \frac{\mu'}{6}\tilde{R}_{\mu 0}^{-1}q\delta_0 - \frac{\mu' a^2}{12}\tilde{R}_{\mu 0}^{-1}\ln\frac{a}{\xi}q\delta_0\lambda_0\right)U_{\theta,1}^{(0)} = 0$$

Sherzod ISROILOV, Ikrom AMIROV, Khudoynazar AKHATOV

$R_{\mu m}$ – the viscoelastic operator of the material layer $\vec{V}$ - velocity vector of a particle of fluid, V_i $(i = r, \theta, z)$ -its components, p_{ij} $(i, j = r, \theta, z)$ - stress tensor components in the fluid, ρ_0' - density of a liquid at rest, p - induced fluid pressure, v' - kinematic viscosity coefficient, $\mu' = \rho_0' v'$ - coefficient of viscosity.

$$\lambda_n^m = \left[\frac{1}{b_n^2} \left(\frac{\partial^2}{\partial t^2} \right) - \frac{\partial^2}{\partial z^2} \right]^m, \quad b_n^2 = \frac{\mu_n}{\rho_n}, \quad n = 0, 1, 2; \quad m = 0, 1. \quad (2)$$

Here b_n --n is the speed of transverse wave propagation in the nth layer Now we transfer equation (1) to dimensionless coordinates. For this, we replace the main parameters as follows

$$r_1 = h \cdot r_1^*, r_2 = h \cdot r_2^*, a = h \cdot a^*, b = h \cdot b^*, \xi = h \cdot \xi^*, t = \frac{l}{b_0} \cdot t^*, z = l \cdot z^*,$$

$$b_1 = b_0 \cdot b_1^*, b_2 = b_0 \cdot b_2^*, U_{\theta,0}^{(0)} = U_{\theta,0}^{*(0)}, U_{\theta,1}^{(0)} = h \cdot U_{\theta,1}^{*(0)}, \rho_0 = \rho_0^* \frac{\mu}{b_0^2}, \mu' = \mu'^* \frac{\mu}{b_0} h, v' = v'^* \cdot b_0 \cdot h$$

(3)

We transfer the system of equations to dimensionless coordinates using expressions (Farshidianfar, Oliazadeh, 2012). For ease of notation, we drop the indices (*) of dimensionless parameters and get the following system. (1) the solution of the system of equations

$$U_{\theta,0}^{(0)} = H_1 e^{i(kz - \omega t)} \qquad \text{va} \qquad U_{\theta,0}^{(1)} = H_2 e^{i(kz - \omega t)}. \qquad (4)$$

we look for it. Here k- wave number, ω - circular frequency of vibrations

$$\frac{a^2}{r_1^2} \left((1 - N_1(\omega))^2 - \frac{r_1^2 - a^2}{12} \left(\frac{\xi}{l} \right)^2 (1 - N_1(\omega)) \left[-d_1 \omega^2 + k^2 (1 - N_1(\omega)) \right] - \right.$$

$$-\frac{r_1^2\left(r_1^2-a^2\right)}{144}\left(\frac{\xi}{l}\right)^4\left[-d_1\omega^2+k^2\left(1-N_1(\omega)\right)\right]^2\right\}\left\{\frac{r_1^2}{4}\left(\frac{\xi}{l}\right)^2\left(1-N_0(\omega)\right)\left[-\omega^2+k^2\left(1-N_0(\omega)\right)\right]U_{\theta,0}^{(0)}+$$

$$+\left[\frac{1}{2}\left(\left(\frac{\xi}{l}\right)^2\left(1-N_0(\omega)\right)\left[-\omega^2+k^2\left(1-N_0(\omega)\right)\right]-\frac{4}{r_1^2}\left(1-N_0(\omega)\right)^2\right]+$$

(5)

$$+\frac{r_1^2}{8}\left(\ln\frac{r_1}{\xi}-\frac{1}{4}\right)\left(\frac{\xi}{l}\right)^4\left[-\omega^2+k^2\left(1-N_0(\omega)\right)\right]^2\right]U_{\theta,0}^{(1)}\right\}=0,$$

$$\left(\frac{a^2}{4}\lambda_0-\frac{\mu'a^2}{6}q\delta_0\right)U_{\theta,0}^{(0)}+\xi\left(\frac{1}{2}(\lambda_0-\frac{4}{a^2})+\frac{a^2}{8}(\ln\frac{a}{\xi}-\frac{1}{4})\lambda_0^2-\frac{\mu'}{6}\tilde{R}_{\mu0}^{-1}q\delta_0-\frac{\mu'a^2}{12}\tilde{R}_{\mu0}^{-1}\ln\frac{a}{\xi}q\delta_0\lambda_0\right)U_{\theta,1}^{(0)}=0$$

$v_0^{(i)}$ --the main parts of the torsional displacement of the middle layer of the shell, h-the middle thick layer, Putting these into equations (5), after some implifications, we get the following frequency equation.

$$\left[-a_{21}\omega^6-a_{22}\omega^4k^2-a_{23}\omega^2k^4-a_{24}k^6+a_{25}\omega^4+a_{26}\omega^2k^2+a_{27}k^4-a_{28}\omega^2-a_{29}\omega^2\right]C_1+$$

$$+\left[b_{21}\omega^8+b_{22}\omega^6k^2+b_{23}\omega^2k^6+b_{24}\omega^4k^4+b_{25}k^8-b_{26}\omega^6-b_{27}\omega^4k^2-b_{28}\omega^2k^4-b_{29}k^6+b_{30}\omega^4+$$

$$+b_{31}\omega^2k^2+b_{32}k^4-b_{33}\omega^2-b_{34}k^2+b_{35}\right]C_2=0 \qquad (6)$$

$$\left[c_1Ik^3\omega-c_2\omega^2-c_3k^2\right]C_1+\left[d_1Ik^2\omega^3-d_2Ik^4\omega+d_3\omega^4+d_4\omega^2k^2+\right.$$

$$\left.+d_5k^4-d_6Ik^2\omega-d_7\omega^2-d_8k^2+d_9\right]C_2=0$$

(6) we can determine the expressions before C1 and C2 in the system of equations.

$$A_{11}=-a_{21}\omega^6-a_{22}\omega^4k^2-a_{23}\omega^2k^4-a_{24}k^6+a_{25}\omega^4+a_{26}\omega^2k^2+a_{27}k^4-a_{28}\omega^2-a_{29}\omega^2$$

$$A_{12}=b_{21}\omega^8+b_{22}\omega^6k^2+b_{23}\omega^2k^6+b_{24}\omega^4k^4+b_{25}k^8-b_{26}\omega^6-b_{27}\omega^4k^2-b_{28}\omega^2k^4-b_{29}k^6+b_{30}\omega^4+$$

$$+b_{31}\omega^2k^2+b_{32}k^4-b_{33}\omega^2-b_{34}k^2+b_{35}$$

$$A_{21}=c_1Ik^3\omega-c_2\omega^2-c_3k^2$$

$$A_{22}=d_1Ik^2\omega^3-d_2Ik^4\omega+d_3\omega^4+d_4\omega^2k^2+d_5k^4-d_6Ik^2\omega-d_7\omega^2-d_8k^2+d_9$$

Let's put it instead

$$A_{11}C_1 + A_{12}C_2 = 0$$
$$A_{21}C_1 + A_{22}C_2 = 0$$

We numerically solve the obtained frequency equation (6) using the "Maple 17" application software package and accept the regular viscosity kernel for the outer layer in this form

$$K_0(t) = \sum_{n=1}^{\infty} \frac{\alpha_n}{\tau_n} e^{-\frac{t}{\tau_n}}, \qquad \sum_{n=1}^{\infty} \alpha_n = 1, \tag{7}$$

Here α_n - viscoelastic parameters of the middle layer; τ_n - relaxation times. We get $N_0(\omega)$ the appearance of the core (15) by putting it in formula (2)

$$N_0(\omega) = \sum_{n=1}^{\infty} \frac{\alpha_n(1 - i\omega\tau_n)}{1 + \omega^2\tau_n^2} . \tag{8}$$

$\alpha_1 = 0{,}6; \alpha_2 = 0{,}3; \alpha_3 = 0{,}1; \tau_1 = 0.16 \cdot 10^5 \, \textit{мкс}; \tau_2 = 0.14 \cdot 10^7 \, \textit{мкс}; \tau_3 = 0.11 \cdot 10^8 \, \textit{мкс}.$

Physical and mechanical characteristics of the outer layer as a material

EDM-6 $E = 3{,}6 \cdot 10^8 \, Pa; \rho = 128 kg/m^3; \nu = 0.36.$ - we accept substances.
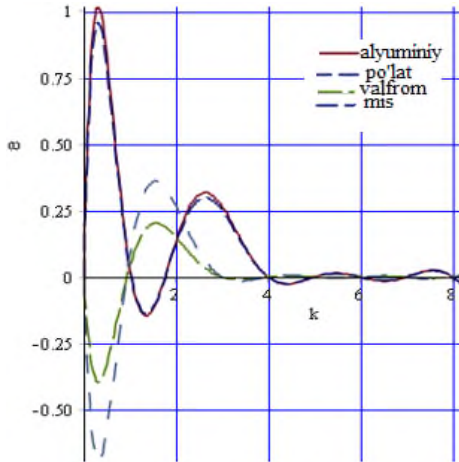
Figure 2. The relationship between frequency and wave number when the outer layer is viscous-

Figure 2 shows the frequency dependence of the wave number during the torsional vibrations of the non-stationary interaction of a two-layer cylindrical shell with the inner layers of steel, aluminum tungsten, and the outer layer of copper EDM-6 with a viscous fluid . show It follows from the above graphs that the dependence of the vibration frequency on the wave number in a two-layer shell with a thin elastic outer layer is not directly proportional as in a homogeneous elastic shell. We can see that the frequency decreases as the wave number increases.

The Conclusion. Equations of vibration in the non-stationary interaction of a two-layered cylindrical shell with a viscous-fluid allow to determine the relationship between frequency and wave number.

$\omega \sim k$ connection has a different character at different values of thicknesses and lengths of the middle layer of the cylindrical shell. At fixed values of the wave number, increasing the thickness or length does not lead to an increase in the frequency of vibration as in the elastic case.

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